CS 370 Fall 2008: Assignment 2

Instructor: Professor Keith Geddes

Lectures: MWF 3:30-4:20 MC 2017

Web Site: UW-ACE

Due: Thu Oct 9, 2008, 5:00 pm, in the Assignment Boxes, 3rd Floor MC

Note: Read the guidelines for assignments specified on UW-ACE in `Course Information` (`Resources > Course Syllabus`), specifically the link `Procedures for assignments`.

1. For $n = 1, 2,$ and 3, derive the Lagrange form of the polynomial $p_n(x)$ of degree n which interpolates the following data:

$$(x_i, f(x_i)),\ 1 \leq i \leq 4$$

where

$$f(x) = \sin(e^x - 2)$$

and where

$$x_1 = 0.6,\ x_2 = 0.7,\ x_3 = 0.8,\ x_4 = 1.0\,.$$

For each n, calculate the resulting estimate for the value of the function at $x = 0.9$ and calculate its absolute error.

2. Consider the following alternative representation for a cubic spline $S(x)$:

$$S(x) = \begin{cases} a + b(x-1) + c(x-1)^2 - \frac{1}{4}(x-1)^2(x-2) & 1 \leq x \leq 2 \\ \\ e + f(x-2) + g(x-2)^2 + \frac{1}{4}(x-2)^2(x-3) & 2 \leq x \leq 3. \end{cases}$$

We also wish our cubic spline to satisfy the boundary conditions:

$$\frac{d^2S}{dx^2}(1) = 0\ ,\ \ \frac{d^2S}{dx^2}(3) = 0\ .$$

(a) What are the conditions on the coefficients a through g such that $S(x)$ interpolates the points (1,1), (2,1), and (3,0)? Deduce the values of a and e.

(b) What is the condition on the coefficients such that $S'(x)$ is continuous at $x = 2$?

(c) Show that enforcing the boundary conditions at $x = 1$ and $x = 3$ leads to $c = -\frac{1}{4}$ and $g = -\frac{1}{2}$.

(d) Compute the values of b and f from part (a).

(e) To ensure that $S(x)$ is a cubic spline, what other condition needs to be checked? (It is not necessary to actually verify this condition for the purpose of this exercise.)

3. Write a Matlab function called `MySpline` which reads in a set of `x` and `y` values (each as vectors), and outputs three vectors of coefficients, `a`, `b`, and `c`, corresponding to the **natural cubic spline** that passes through the given points. The function prototype is

```
[a, b, c] = MySpline(x, y) .
```

Skeleton code for this function, including documentation on how to use the function, is supplied. The output coefficients correspond to the parameters for the special representation of the cubic spline mentioned in the Course Notes; namely, for $i = 1, \ldots, n-1$,

$$p_i(x) = a_{i-1}\frac{(x_{i+1}-x)^3}{6h_i} + a_i\frac{(x-x_i)^3}{6h_i} + b_i(x_{i+1}-x) + c_i(x-x_i) \ , \qquad (1)$$

where $h_i = x_{i+1} - x_i$. Note that since all Matlab arrays start at index 1, the vector of `a`-values is offset by one.

You may use the supplied Matlab function `EvaluateMySpline` to make sure your spline function is working correctly. At a Matlab prompt, type "`help EvaluateMySpline`" to see more details on this. You might also find the script `a2q3_test.m` useful to test your function.

4. **Graphics display of ampersand.**

Create a parametric curve representation of an ampersand (shift-7 on a standard keyboard) in your handwriting. The representation should be based on smooth parametric curve interpolation as described in §3.3 of the Course Notes.

Squared graph paper is useful for writing the ampersand and determining the initial data points.

What to do:

Write a Matlab command file named `InitAmp.m` to:

1. initialize data arrays for the crude initial shape data, and parameter values;
2. compute `ppform` for the cubic spline interpolation of this data as a parametric curve.

Prepare three Matlab .m files, one for each of the following plots.

1 A plot with grid lines and the interpolation data for the crude initial shape, plotted using the symbol 'X' (not a piecewise linear interpolating curve).
Input is the initialized arrays of `InitAmp.m` from step 1.

2 A smooth plot of an ampersand created by refining the parameter partition by a factor of 3. Plot this without axes. Input is `ppform` and the parameter array from `InitAmp.m` .

3 A plot showing a smoother version of the ampersand overlaid on the simple piecewise linear interpolating curve of the original data. You may pick any refinement of the parameter partition that gives a good smoothing. Do not use any axes or grid lines for this plot.

Note: You should always label and title your plots as appropriate.